An Approximate Model of Vortex Decay in the Atmosphere

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An approximate analysis of atmospheric effects on wake vortex motion and decay is presented. The effects of density stratification, turbulence, and Reynolds number are combined in a single model so that the relative importance of different parameters can be estimated. Predicted wake motion is shown to be in good agreement with limited data from ground facility and flight test measurements taken under low-turbulence conditions. Wake decay was found to depend strongly on both density stratification and turbulence. For typical levels of turbulence, wake decay was found to result from the Crow instability except under strongly stratified conditions.

Nomenclature

A = area of wake oval
$\mathcal{R}$ = wing aspect ratio
B-V = Brunt-Väisälä
b = vortex spacing
C_D = viscous force coefficient, viscous force/$(\rho V^2/2)L$
C_L = wing lift coefficient, lift/$[(\rho U^2/2)(S^2/\mathcal{R})]$
c = speed of sound, $(\gamma R\theta)^{1/2}$
g = acceleration due to gravity, 9.81 m/s^2
H = dimensionless wake descent distance, z/b
I = wake impulse per unit length
L = width of wake oval, 2.09 b
N = Brunt-Väisälä frequency,

$$\left(\frac{g}{\iota}\frac{d\rho}{dz}-\frac{g^2}{c^2}\right)^{1/2} \text{ (note definition of } z)$$

NS = dimensionless stratification parameter, $NS = Nb/V_0$
P = Brunt-Väisälä period, $2\pi/N$
p = pressure
QS = dimensionless turbulence parameter, q/V_0
q = root-mean-square turbulence velocity
R = gas constant
R_e = Reynolds number, $\rho VL/\mu$
S = wing span
s = variable in density function, $\rho=\rho(p,\theta,s)$
T = dimensionless time, tV_0/b
T_L = dimensionless time for vortex pair linking, $t_L V_0/b$
t = time
t_L = time for vortex linking due to "Crow instability," s
U = aircraft forward speed
V = wake descent speed
V_0 = initial wake descent speed
z = wake vertical coordinate or descent distance, defined positive downward
β = temperature coefficient of volume expansion, $-(1/\rho)(d\rho/d\theta)$
Γ = vortex circulation
Γ_0 = initial value of vortex circulation
γ = ratio of specific heats
ϵ = turbulent energy dissipation rate per unit mass
ζ = coefficient of volume expansion for the variable s, $-(1/\rho)(d\rho/ds)$
η = dimensionless turbulence parameter, $(b\epsilon)^{1/3}/V_0$
θ = temperature
Λ = turbulence scale parameter
μ = coefficient of viscosity
ρ = density

Subscript

A = evaluated for adiabatic conditions

Introduction

THE aircraft trailing wake hazard is well documented and has been the subject of intensive research by both NASA and the FAA for more than a decade. NASA research has focused on wake alleviation through changes in aircraft design, while the FAA has concentrated on operational strategies for minimizing the impact of the wake hazard on airport capacity. Althugh there has been progress in both the NASA and FAA programs, completely satisfactory solutions have not evolved.

A recurring problem has been the determination of the effects of the atmosphere on wake motion and decay. The wake hazard has been observed to exist, under calm atmospheric conditions, for distances significantly greater than FAA-mandated aircraft separations[1] and to persist longer than would be expected based on ground facility measurements.[2] Wake decay measurements,[3-5] over a wide range of conditions indicate that wake decay can be dominated by atmospheric effects.

Research on atmospheric effects has been reviewed in recent years by Lissaman et al.,[6] Donaldson et al.,[7] and Widnall.[8] The effect of density statification has been studied by a number of researchers[9-18] with conflicting results. The disparity in analytical predictions is in sharp contrast to the experimental observations, which uniformly indicate that stable stratification inhibits wake descent.

Hecht et al.[19] have presented a numerical analysis, combining both turbulence and stratification effects, that compares favorably with experimental observations[20] of wake decay in a strongly stratified atmosphere. However, the calculations in Ref. 19 to determine the effect of vortex core size on turbulence generation and wake decay appear to be contradicted by experimental data.[21] The relative effects of stratification and turbulence are difficult to separate, due both to the complexity of the numerical model and the lack of well-documented experimental data with known turbulence effects. However, the favorable agreement of the wake decay results in Ref. 19 appears to result largely from the inclusion of stratification effects.

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The purpose of this paper is to present an approximate analysis that has been used to predict wake motion and decay characteristics in both ground facilities and the atmosphere. The essential physics of turbulence and stratification are included semi-empirically in a consistent manner such that the relative importance of the atmospheric parameters can be estimated. The effects of ground proximity and crosswind shear are not considered.

Wake Decay Model

An aircraft develops lift by imparting momentum to a region of fluid that may be loosely called its wake. This wake typically rolls up into two regions of concentrated vorticity that can, for many purposes, be approximated by a pair of point vortices. This approximation may be justified based on observed decay characteristics except near the ground, in cross winds, or during the later stages of wake decay. If the Boussinesq approximation is made, the wake momentum or impulse per unit length may still be defined as $\rho b \Gamma$. If it is further assumed that the vortices maintain a constant spacing as they descend (a phenomenon also observed to be approximately true), a relatively simple but useful model results.[14] The flowfield is characterized by an oval region of fluid descending at a velocity directly proportional to the circulation, $V=\Gamma/2\pi b$. As the wake descends through the atmosphere, it may experience viscous and buoyancy forces that reduce the impulse. By calculating these forces, which determine the rate of change of impulse, the circulation, velocity, and position of the wake can be determined.

Viscous Interaction

As the roughly oval-shaped wake descends through the atmosphere, it is subject to laminar and turbulent viscous forces. An empirical estimate of their magnitude was made by assuming that the viscous forces acting on the wake shown on the left side of Fig. 1 would be the same as those acting on a similar solid body and in particular the solid body shown on the right side of the figure. With this assumption, the viscous force per unit wake length can be written

$$\text{viscous force} = -\frac{\rho V^2}{2} C_D L \tag{1}$$

The solid-body viscous force coefficient, based on the descent velocity and L, was taken from Ref. 22 and is 0.2 for $R_e > 600{,}000$ and 1.4 for $R_e < 400{,}000$. Since there is no a priori justification for this assumption, its validity must be evaluated experimentally.

Stratification Effects

As the wake descends through the atmosphere, it is compressed due to the change in ambient pressure with altitude. This compression raises the temperature of the fluid in the oval at an essentially adiabatic rate. If the lapse rate in the surrounding atmosphere is adiabatic, then the density inside and outside the wake oval are the same. If the lapse rate is not adiabatic, there will be a difference in the density and therefore a net buoyancy force on the fluid in the oval.

A parameter indicative of the degree of stratification in a fluid is the buoyancy or Brunt-Väisälä (B-V) frequency N. N is the hypothetical frequency of oscillation of a fluid "particle" displaced from an equilibrium position, i.e., the natural frequency of the vertical buoyancy oscillation. It can be expressed as

$$N=\left(\frac{g}{\rho}\frac{d\rho}{dz}-\frac{g^2}{c^2}\right)^{1/2} \tag{2}$$

In general, the density is a function of the pressure, temperature, and other variables such as the humidity (air) or salinity (water). If $\rho=\rho(p,\theta,s)$, then

$$N=\left[-g\beta\left(\frac{d\theta}{dz}-\frac{d\theta}{dz}\bigg|_A\right)-g\zeta\left(\frac{ds}{dz}-\frac{ds}{dz}\bigg|_A\right)\right]^{1/2} \tag{3}$$

where

$$\beta=-\frac{1}{\rho}\frac{d\rho}{d\theta} \quad \text{and} \quad \zeta=-\frac{1}{\rho}\frac{d\rho}{ds}$$

In the atmosphere, humidity gradients can make a small but significant contribution to stratification, particularly if the atmosphere is "layered." For a perfect gas (dry air), $\rho=\rho(p,\theta)$ and $\beta=1/\theta$. The Brunt-Väisälä frequency is then expressed as

$$N=\left[-\frac{g}{\theta}\left(\frac{d\theta}{dz}-\frac{d\theta}{dz}\bigg|_A\right)\right]^{1/2} \tag{4}$$

For water, which is essentially incompressible, the adiabatic gradients are very small and the Brunt-Väisälä frequency

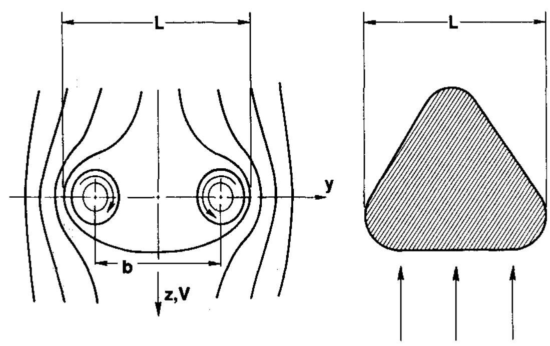
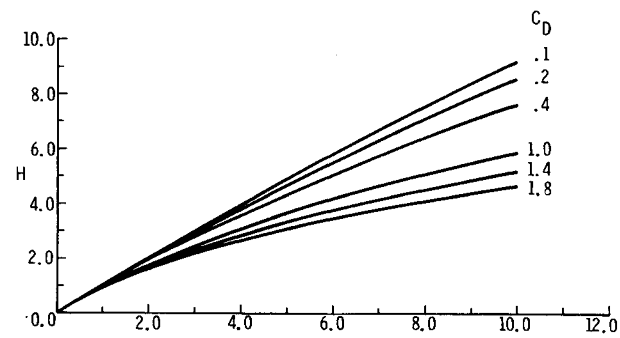
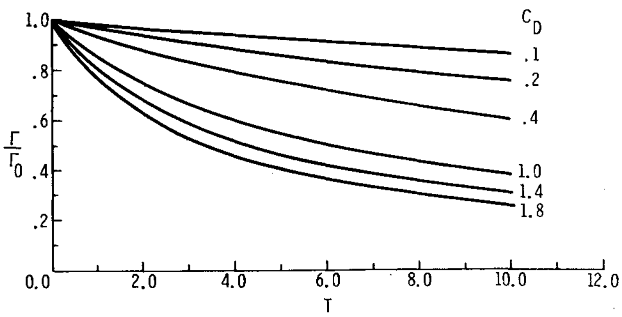


Fig. 1 Descending wake and solid body that are assumed to experience equivalent viscous forces.



a) Wake descent.



b) Circulation decay.

Fig. 2 Effect of varying C_D on wake motion and decay with $NS=0$, $QS=0$.

becomes

$$N=\left(-g\beta\frac{\mathrm{d}\theta}{\mathrm{d}z}-g\zeta\frac{\mathrm{d}s}{\mathrm{d}z}\right)^{1/2} \tag{5}$$

Thus, in water, density stratification can be achieved through temperature gradients, salinity gradients, or both.

In order to have dynamic similarity between full-scale flight conditions and model test conditons, Lissaman et al.[6] pointed out that the ratio of two characteristic times should be the same. One time is proportional to B-V period, $P=2\pi/N$ and the other is proportional to the time b/V_0 required for the wake to descend one vortex spacing in an inviscid, unstratified fluid. Although various definitions of the similarity ratio have been used, all are simply related to b/V_0P. The parameter used herein is

$$NS=N\,b/V_0 \tag{6}$$

This can be written in terms of aircraft parameters as

$$NS=N\frac{4\pi S(b/S)^3 Æ}{C_L U} \tag{7}$$

since

$$b/V_0=2\pi b^2/\Gamma_0 \text{ and } \Gamma_0=C_L US^2/2bÆ$$

The buoyancy force per unit length acting on the wake is $gA\Delta\rho$ where $\Delta\rho$ is the difference in density between the inside and outside of the wake oval and A the wake oval area, which is assumed to be a constant equal to $[\pi(1.73)\times(2.09)b^2]/4$. The buoyancy force can be written in terms of the classical (B-V) buoyancy frequency as

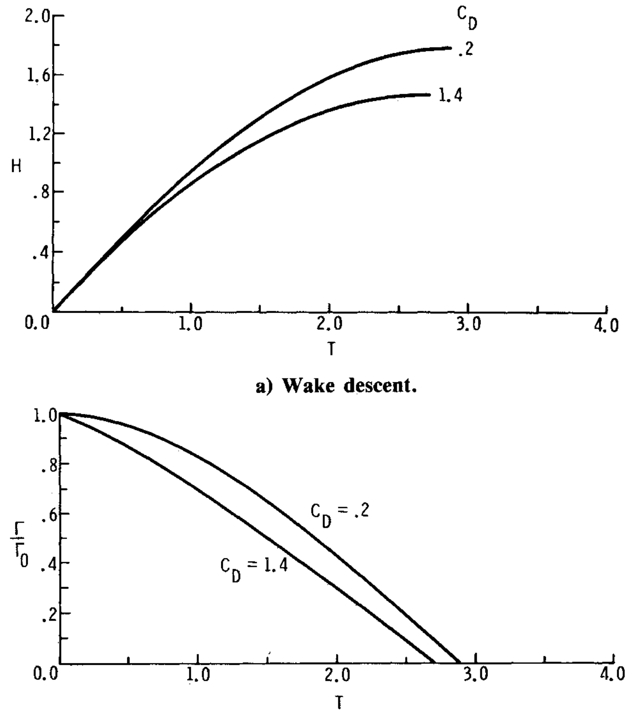
$$\text{buoyancy force}=-\rho AN^2 z \tag{8}$$

It should be noted that the predicted oscillation frequency for a wake will always be lower than the classical value N, since the impulse of a wake is always greater than the momentum of an equal-area fluid "particle" moving at the same speed.

Wake decay is assumed to occur at the time when the impulse and descent velocity reach zero, which for buoyancy effects alone occurs at a time equal to one-fourth of the oscillation period. Even though there may still be a strong vortex core flow, the circulation is greatly reduced (zero in the present model) and should not pose a hazard to large aircraft having a wing span much larger than the core diameter. In addition, the assumption that the vorticity is concentrated in a small core is no longer valid in the late stages of decay, particularly if the wake begins to rise again under buoyancy forces. This may be understood by referring to the analysis attributed to Yates in Ref. 7 for the rate of change of the polar moment of vorticity. Since the polar moment can be used to infer how widely vorticity is spread in the core, it is a quantitative indicator of the validity of the concentrated vorticity assumption. There are two processes that cause the polar moment to change. The obvious one is a viscous effect that tends to spread vorticity throughout the wake oval. The other is due to a vertical asymmetry in the pressure or vorticity field. If a wake is descending, the asymmetry due to the vorticity detrained from the oval will tend to concentrate the remaining vorticity in the core. If the Reynolds number is high, this can apparently offset or nearly offset the viscous diffusion, resulting in wake decay from the "outside in." If a buoyant vortex begins to rise, then both viscous effects and asymmetry effects act together to reduce the concentration of vorticity and rapidly enlarge the core.

Atmospheric Turbulence Effects

In addition to the viscous interactions occurring as the wake descends through a quiescent atmosphere, turbulence can play a significant role in enhancing wake decay. Donaldson et al.[7] presented an approximate analysis that indicated that even mild turbulence could arrest wake descent due to turbulent transport of vorticity across streamlines. For constant vortex spacing, the rate of change of circulation was given as



b) Circulation decay.

Fig. 3 Effect of varying C_D with $NS=0.8$, $QS=0$.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t}=-0.82\frac{q\Gamma}{b} \tag{9}$$

The turbulence-generated viscous force changes the impulse at a rate

$$\frac{\mathrm{d}I}{\mathrm{d}t}=\rho b\frac{\mathrm{d}\Gamma}{\mathrm{d}t}=-1.64\pi\rho bqV \tag{10}$$

or in a form analogous to the other viscous forces

$$\frac{\mathrm{d}I}{\mathrm{d}t}=\left(\frac{\rho V^2}{2}\right)\left(4.93\frac{q}{V}\right)L \tag{11}$$

The factor 4.93 q/V can be considered a variable turbulence force coefficient that becomes increasingly important as the wake descent velocity decreases. The constant 0.82 was considered a "best guess" in Ref. 7 and therefore is subject to some uncertainty. It is used in lieu of a better value and is subject to experimental verification.

Motion and Decay Equations

The basic equation for wake motion and decay is that the rate of change of impulse per unit length of wake is equal to the sum of the forces acting on a unit length wake given by Eqs. (1), (8), and (11). In dimensional form, this can be written as

$$\frac{\mathrm{d}I}{\mathrm{d}t}=2\pi\rho b^2\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{\rho V^2}{2}C_D L-\rho AN^2 z-\frac{\rho V^2}{2}\,4.93\frac{q}{V}L \tag{12}$$

Introducing the nondimensional parameters $H=z/b$, $T=tV_0/b$, $NS=Nb/V_0$, and $QS=q/V_0$, one obtains

$dH/dT = V/V_0 = \Gamma/\Gamma_0$ and

$$\frac{d^2H}{dT^2} + \frac{C_D L}{4\pi b}\left(\frac{dH}{dT}\right)^2 + 0.82\ QS\left(\frac{dH}{dT}\right) + \frac{A(NS)^2}{2\pi b^2}H = 0 \tag{13}$$

This relatively simple equation can be solved to predict both wake motion and circulation decay. Although the equation is easily solved numerically, many characteristics of wake motion and decay can be evaluated analytically. In particular, when the turbulence and stratification terms are large enough that the nonlinear term can be neglected, well-known analytic solutions are available.

Crow Instability

Vortex pair linking, known as the Crow instability, has been studied by a number of researchers[23-27] and is relatively well understood. The time at which linking occurs is often taken to be the wake lifetime. In Ref. 26, a composite time-to-link function was developed by considering the cases of both very strong and very weak turbulence and was shown to be in reasonable agreement with the limited available published data. However, the data scatter was large and the maximum time-to-link for a given value of the dimensionless turbulence intensity η was typically underestimated by a factor of two or more.

Since a conservative estimate of wake lifetime was the primary interest in the present analysis, particularly under low-turbulence conditions, the time-to-link estimate obtained for strong turbulence, $T_L = 0.41/\eta$, was used for all turbulence conditions.

In order to have a consistent wake decay model, η was defined in terms of the turbulence parameter QS. An approximation given in Ref. 7 is $q = 2\Lambda^{1/3}\epsilon^{1/3}$, where Λ is approximately equal to one-half of the longitudinal integral scale of the turbulence. The dimensionless time-to-link is then

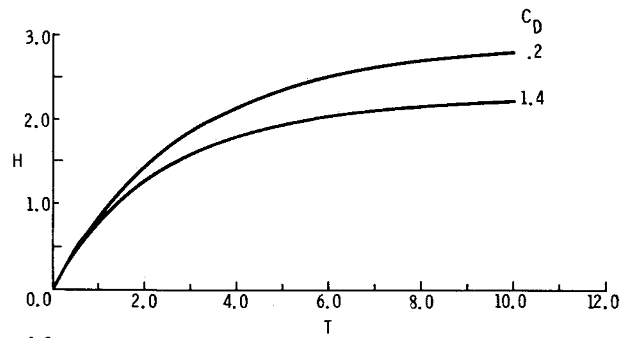
$$T_L \approx \frac{0.8\Lambda^{1/3}}{QSb^{1/3}} \tag{14}$$

The integral scale and Λ depend strongly on atmospheric stability and height above the ground. However, since T_L is proportional to $\Lambda^{1/3}$, only the correct order of magnitude of Λ needs to be known to estimate T_L within about a factor of two. Because of this insensitivity and the resulting simplification, $\Lambda = 8b$ is assumed for all conditions. This is admittedly crude, but $b < \Lambda < 8b$ for many aircraft and atmospheric conditions and therefore $\Lambda = 8b$ should provide a conservative estimate of T_L. With this assumption,

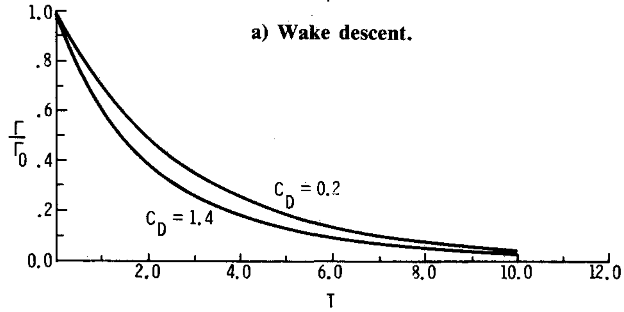
$$T_L = 1.6/QS \tag{15}$$

Results and Discussion

Since the vortex decay model is relatively simple and contains several constants whose values are not known precisely, the first part of the results explores the relative importance of the various terms in the decay equation and presents an abbreviated evaluation of the model using available wake trajectory data. Circulation decay data of a known accuracy are more difficult to obtain, particularly over a range of atmospheric conditions. Therefore, an evaluation of the calculated decay prediction accuracy has not been made, although the trends appear to be qualitatively correct. The second part of the results is a parametric application of the model to predict the wake motion and decay over the range of atmospheric parameters commonly encountered. The range of parameters was chosen to show how the state of the atmosphere determines the wake lifetime and to identify those conditions when the wake decay is sufficiently rapid to allow reduced aircraft separations in the terminal area and those conditions that might have an extended hazard duration.

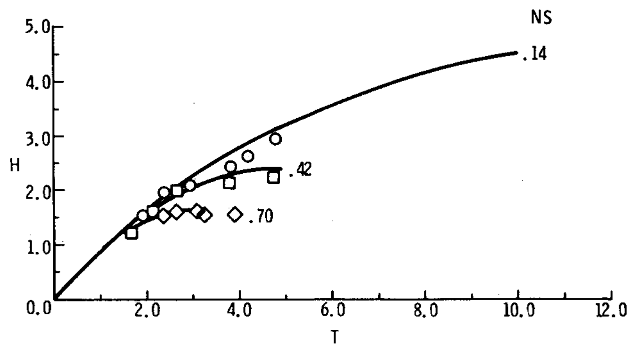


a) Wake descent.

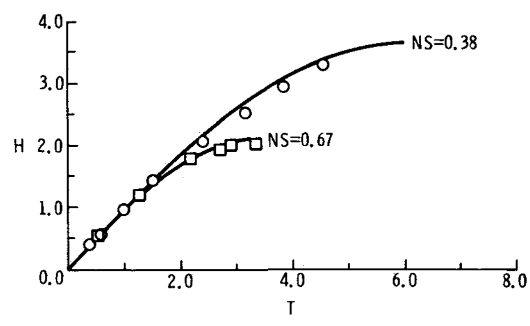


b) Circulation decay.

Fig. 4 Effect of varying C_D with $NS = 0$, $QS = 0.4$.



a) Ground facility data.



b) Flight test data.

Fig. 5 Comparison of measured and predicted wake motion for low-turbulence conditions ($QS = 0$).

Effects of Model Parameters

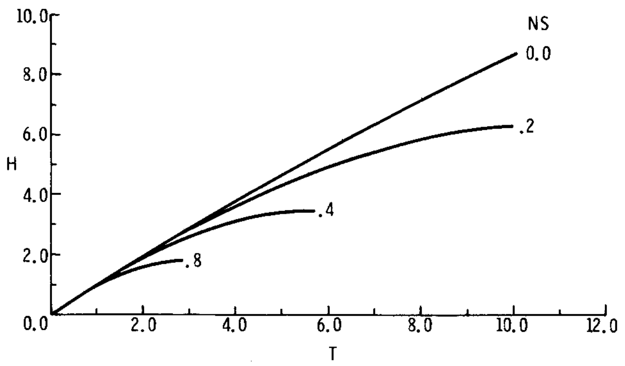
Figure 2 shows the results of varying the viscous interaction coefficient C_D about the values chosen based on the solid-body analogy. Figure 2a presents the dimensionless descent distance H as a function of dimensionless time T and Fig. 2b the normalized circulation Γ/Γ_0 as a function of T. As shown, there is a significant effect due to varying C_D for both the descent distance and circulation histories. The effect of varying C_D about the chosen values of 0.2 and 1.4 is less significant than that due to changing C_D from 0.2 to 1.4. Thus, the assumption that the viscous coefficient varies with Reynolds number in a manner analogous to that of a solid-body flowfield may be more critical than the absolute level of the coefficient at a given Reynolds number. The predicted differences provide a possible explanation for the decay trend differences, noted in Ref. 2, between ground facility and flight test data.

Figures 3 and 4 show the effects of strong stratification and turbulence on the wake trajectory and decay for two values of C_D or Reynolds number effect is much smaller representing the assumed Reynolds number effect. The C_D than that of strong stratification or turbulence, as is evident by comparing Fig. 2 with either Fig. 3 or 4. For smaller values of turbulence and/or stratification, the wake trajectory and decay characteristics will depend on all of the parameters in the model and may not be dominated by any single parameter.

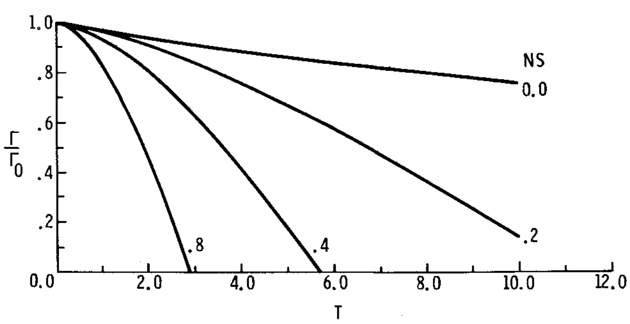
In order to validate this model, a comprehensive set of data is required, consisting of both vortex motion and decay measurements and measurements that can be used to estimate the stratification and turbulence characteristics of the atmosphere. In general, data of this type do not exist. There are several sets of data that at least partially meet these criteria—the flight data reported in Refs. 20 and 28 and ground facility measurements of Refs. 4 and 5. The most difficult task in defining either the facility or flight environment is determining the level and scale of turbulence. Measurements of circulation decay are also very difficult, due to the requirement of measuring small mean velocities that may be of the same order as the turbulent fluctuations. Therefore, wake trajectory data, which are relatively easy to measure, have been used to evaluate the model for a range of stratification for both ground and flight data. It should be recognized that this is not a complete validation of the model, since the effects of turbulence have not been checked in a direct way or are not known for either the ground or flight data. However, the ground facility data were chosen because the turbulence levels were "low" in an unquantified sense.

Figure 5 shows the wake trajectory comparisons for a range of stratification parameters. Fig. 5a shows trajectory comparisons for a swept, tapered wing of aspect ratio 7.8 from Ref. 5. The three stratification levels correspond to levels typically encountered in the atmosphere. The circle symbols, for $NS = 0.14$, end prior to the decay of the wake as, under these conditions, the wake descended below the region of flow visualization. A C_D of 1.4, chosen a priori based on the Reynolds number, and a zero turbulence level were used for the calculation. The turbulence characteristics in the facility are quantitatively unknown. The dominant effect of stratification is clearly predicted and the absolute agreement is very good, considering the approximate nature of the analysis and the difficulty of measuring and controlling the facility environment.

Figure 5b shows similar comparisons for two sets of flight test data. The $NS = 0.38$ data are from tests of a small (11 m span) aircraft described in Ref. 28 and the $NS = 0.67$ data are from tests using a large (60 m span) jet transport described in Ref. 20. The stratification parameters used in the calculations were based on the stated temperature gradients and a zero turbulence level was assumed. A C_D of 0.2 was used, based on the higher wake Reynolds number. The turbulence intensity, which is an input to the calculation, was not measured. However, the turbulence dissipation rate ϵ was estimated and, with an assumption for the turbulence scale, could be used to estimate the turbulence intensity. The estimated value of $(\epsilon)^{1/3}$ for the $NS = 0.38$ case was 0.3 $cm^{2/3}/s$ and for the $NS = 0.67$ case was 1.4 $cm^{2/3}/s$, corresponding to what would be called "negligible" and "light" turbulence, respectively.[28] Including any estimate of the turbulence in the calculation would tend to make the agreement less favorable and perhaps give a more realistic picture of the degree to which the atmospheric effects can be predicted. However, the agreement may also be equally limited by the uncertainty in determining the values of atmospheric parameters to use in the calculation, particularly in estimating the turbulence scale in a stratified atmosphere. The next subsection is

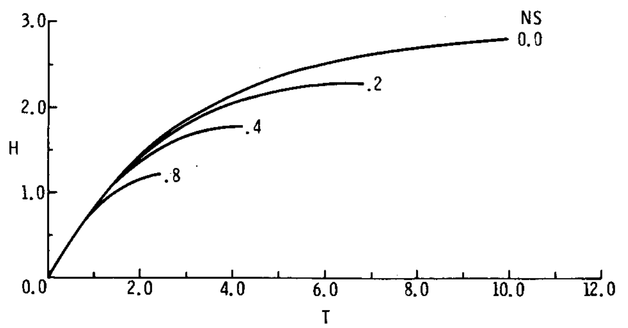


a) Wake descent.

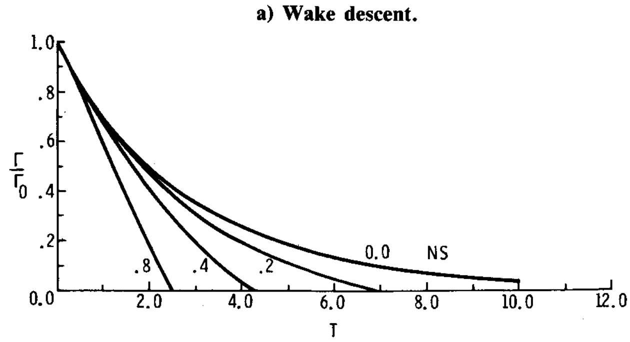


b) Circulation decay.

Fig. 6 Predicted effect of density stratification on wake motion and decay, $QS = 0$.



a) Wake descent.



b) Circulation decay.

Fig. 7 Predicted effect of density stratification, $QS = 0.4$.

a parametric study showing the sensitivity of wake motion and decay to various combinations of stratification and turbulence.

Predicted Wake Decay Characteristics

As discussed in earlier sections, wake lifetime is sensitive to small variations in atmospheric conditions. These variations are so small that their importance has often been overlooked in wake decay measurements and, therefore, there is relatively little data available for verifying the predicted wake decay trends. This is particularly true for turbulence characteristics in a stratified atmosphere. Therefore, the remaining figures present predicted wake motion and decay characteristics without experimental verification. Although quantitative data are not available, qualitative observations support the general trend predicted by the model. It should be noted that the model contains empirical constants and requires verification of both the individual constants and the possible interactions. All of the remaining figures are for decay in the atmosphere for which the Reynolds number is assumed to be sufficiently high that a C_D of 0.2 should be used.

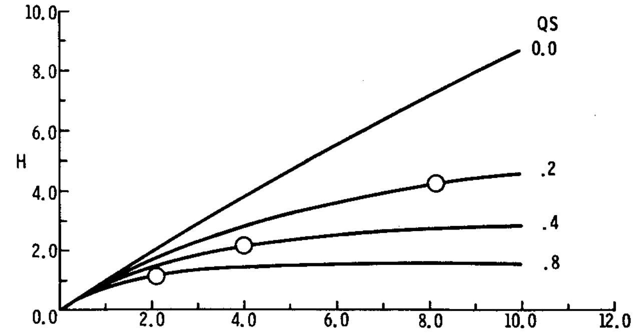
Figure 6 shows wake descent and circulation as a function of T for an NS between 0 and 0.8 and a zero turbulence level. It should be noted that changes in the temperature lapse rate of only a few degrees Celsius per 100 m of altitude are required to produce the predicted variation shown. The occurrence of zero turbulence conditions in the atmosphere is improbable; however, in such conditions, the wake would be expected to persist for a long time for NS less than about 0.2.

The time b/V_0 required for the wake to descend a distance equal to one vortex spacing, which is used for nondimensionalizing both NS and T, ranges from about 10 s for a small general aviation aircraft to about 20 s for a jumbo jet. If one considers the case of a jumbo jet, the NS range of 0-0.8 corresponds to temperature lapse rates ranging from adiabatic to a strong inversion. The "standard atmosphere" lapse rate is just slightly more stable than adiabatic and corresponds approximately to $NS=0.2$. $NS=0.4$ corresponds to a slightly positive (0.25°C/100 m) lapse rate and $NS=0.8$ to about +4°C/100 m. It is important to note that only 1°C/100 m change in lapse rate is required to change NS from 0.4 to 0.2, which would double the predicted wake lifetime for a jumbo jet from 2 to 4 min.

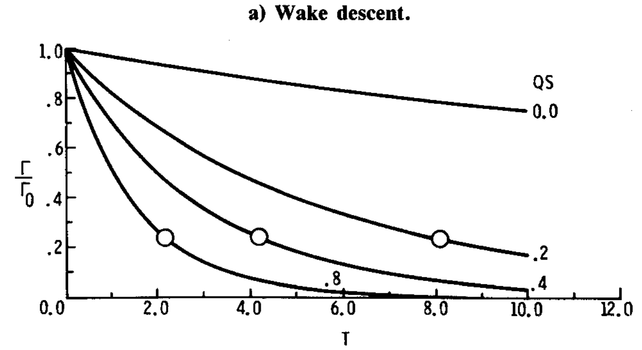
The atmosphere is rarely turbulence free and even small amounts of turbulence can significantly enhance wake decay. Figure 7 shows wake characteristics for the same conditions as Fig. 6 with a turbulence level QS of 0.4. For a large transport aircraft, this would correspond to "light-to-moderate" or "moderate" atmospheric turbulence. Both the maximum wake descent distance and wake lifetime are significantly reduced. Perhaps even more significant is the rapid initial rate of decay of circulation for all levels of stratification. Even for the neutrally stable case ($NS=0$), the circulation is reduced by about 50% at $T=2$.

Due to this sensitivity to atmospheric turbulence, Figures 8-11 show wake motion and decay as a function of the normalized turbulent velocity fluctuation QS for constant stratification. In addition, the predicted occurrence of vortex pair linking due to the Crow instability is indicated by a circle as a relative indication of the importance of different decay mechanisms.

Figures 8 and 9 show the zero and light ($NS=0$ and 0.2) stratification results. The very slow predicted decay for $QS=0$ is shown again to emphasize the persistence of the vortex under conditions of low turbulence and near-neutral atmospheric stability. For $QS=0.2$, which might roughly correspond to the boundary between "negligible" and "light" turbulence[28] for a large transport aircraft, the wake decay is significantly enhanced even though the wake lifetimes are still long. Within the expected accuracy of the model, it would be difficult to predict whether wake demise would result from viscous decay or through the Crow linking process. As the turbulence level increases, Crow linking would be expected to occur sooner and be the dominant decay mode. As discussed earlier, both QS and T are normalized by parameters based on aircraft characteristics. For "light-to-moderate" turbulence, QS might be on the order of 0.4 for a large transport aircraft and 0.8 for a light, general aviation airplane. The predicted occurrence of Crow

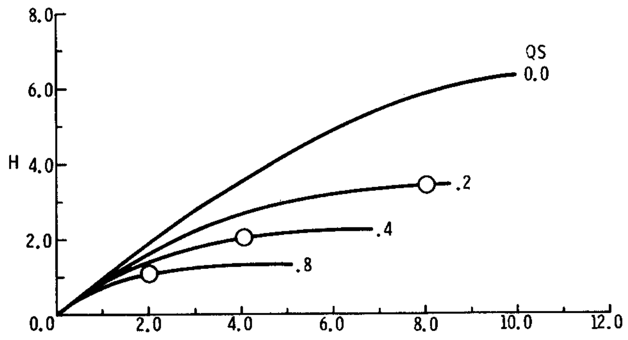


a) Wake descent.

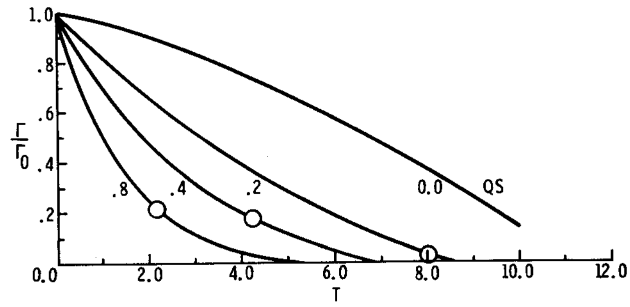


b) Circulation decay.

Fig. 8 Predicted effect of turbulence on wake motion and decay, $NS=0$.

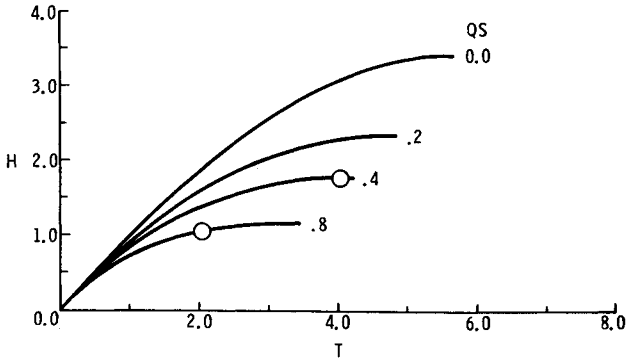


a) Wake descent.

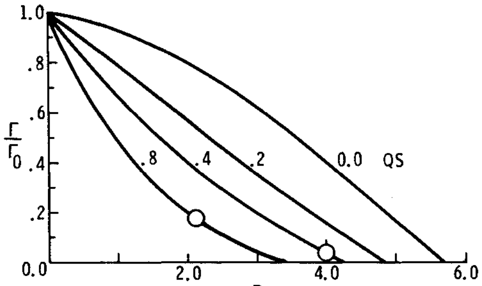


b) Circulation decay.

Fig. 9 Predicted effect of turbulence, $NS=0.2$.

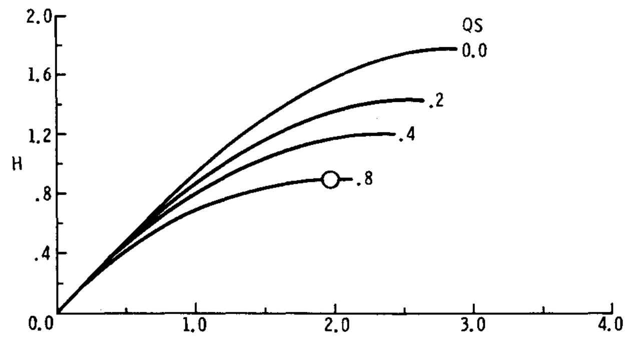


a) Wake descent.

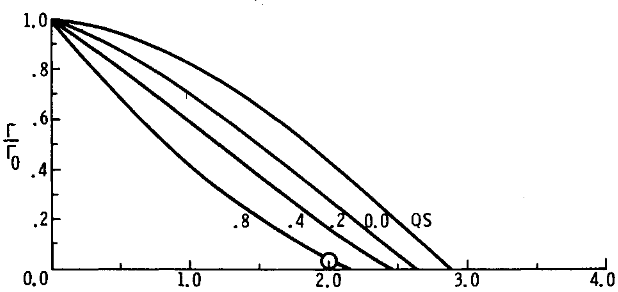


b) Circulation decay.

Fig. 10 Predicted effect of turbulence, $NS = 0.4$.



a) Wake descent.



b) Circulation decay.

Fig. 11 Predicted effect of turbulence, $NS = 0.8$.

linking at $T = 4$ and 2 might correspond to actual linking times of 80 and 20 s, respectively, for the large transport and light plane.

Figures 10 and 11 show wake motion and decay characteristics for a stronger stratification indicating more stable atmospheric conditions. For a large transport aircraft, $NS = 0.4$ would correspond approximately to isothermal atmospheric conditions and $NS = 0.8$ would represent an inversion or increasing temperature with altitude. These conditions produce a decrease in both the maximum wake descent distance and wake lifetime. As a result, the maximum wake lifetimes become less likely to be limited by the Crow instability, except at higher turbulence levels. This trend is consistent with the experimental wake decay observations of Ref. 4, where wake decay was observed to be due to the Crow instability except in strongly stable conditions.

The maximum absolute wake lifetimes predicted for a large transport aircraft in stable conditions range from about 2 min for $NS = 0.4$ down to about 1 min for $NS = 0.8$.

Conclusions

Wake decay in the atmosphere is a complex process that may be strongly influenced by atmospheric parameters which have not been routinely measured in flight test programs. An approximate model has been presented that attempts to account for some of these effects. The model provides a quantitative prediction of the effects of density stratification on wake motion and the results agree well with both ground and flight test measurements. The corresponding predictions of wake decay are in qualitative agreement with wake decay observations.

The turbulence model and the assumption that the effects of turbulence and stratification are linearly additive have not been quantitatively evaluated. The predicted trends and relative importance of the Crow instability are qualitatively correct.

In spite of the limitations inherent in this approximate model, some predicted effects seem quite clear:

1) The effects of Reynolds number on wake decay are generally quite small, but not necessarily unimportant. In the absence of stratification or turbulence effects, Reynolds number effects could at least partially explain the differences in wake lifetimes between ground facility and flight tests.

2) Under conditions of near neutral atmospheric stability and low turbulence, wake lifetimes can exceed the current FAA-mandated aircraft separation times by a factor of about two or more. This is both predicted by the model and observed in flight test measurements.

3) Under conditions of strong stratification, turbulence, or both, wake lifetimes may be less than FAA-mandated separation times by a factor of about two or more.

4) The atmosphere is, on average, both stratified and turbulent. Predictions of wake lifetimes for the "standard atmosphere" with light turbulence are consistent with FAA separation times.

5) The Crow instability may or may not go to completion depending on the relative importance of turbulence and stratification effects.

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